This set contains four pages (beginning with this page)
All questions must be answered
Questions 1 and 2 each weigh 25 % while question 3 weighs 50 %. These weights, however, are only indicative for the overall evaluation.

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# MONETARY ECONOMICS: MACRO ASPECTS
# SOLUTIONS TO AUGUST 18 EXAM

## QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In a simple New-Keynesian model (like in Benigno's 2009 exposition), increased government spending has no effects due to the offsetting decrease in private consumption.

A False. In this model with sticky prices, output is demand determined. Increased public spending will therefore increase demand and production. The upward pressure on prices (relative to long-run prices) will increase the real interest rate and depress private consumption, but not crowd out the fiscal stimulus completely. (The excellent answer may note that an expected increase in future public spending is contractionary as consumers, in anticipation of a lower natural rate of consumption, already reduce consumption now to "smooth" out the effect over time.)

(ii) In the simple cash-in-advance model with only consumption in the utility function, there is no unique optimal monetary policy.

A True. Various monetary policies leading to various inflation rates will have no effect on output and consumption as superneutrality prevails. As only consumption matters for welfare, any inflation rate is as good as the other. So no unique optimal policy exists.

(iii) With economic fluctuations predominantly arising in the money market, Poole's 1970 analysis of optimal operating procedures for monetary policy, suggests that output is best stabilized under a base money operating procedure.

A False. The main point of Poole's analysis is that an interest-rate operating procedure is beneficial for output stabilization if shocks are mainly emanating from the money market, whereas a monetary base operating procedure is best is shocks mainly occurs in the goods market. Indeed, with shocks predominantly occurring in the money market, these shocks can be completely neutralized by using an interest-rate operating procedure.

## QUESTION 2:

### Monetary credibility problems and wage indexation

Consider an economy characterized by the following aggregate supply schedule:

$$y_t = p_t - w_t, \tag{1}$$

where $y_t$ is log of output, $w_t$ is log of nominal wages, and $p_t$ is log of the price level. Nominal wage determination in period $t$ is assumed to be characterized by one-period nominal wage contracts signed in period $t-1$. These contracts can be partly or fully indexed to the actual period-$t$ price level. Nominal wages are therefore assumed to be given by:

$$w_t = (1-\theta)\,\mathrm{E}_{t-1}[p_t] + \theta p_t, \qquad 0 < \theta \leq 1, \tag{2}$$

where $\mathrm{E}_{t-1}[p_t]$ is the (rationally) expected price level and $\theta$ is an indexation parameter.

(i) Derive the economy's aggregate supply schedule when nominal wages are determined according to (2). Interpret the expression.

A Inserting (2) into (1) gives

$$y_t = p_t - (1-\theta)\,\mathrm{E}_{t-1}[p_t] - \theta p_t$$

and

$$y_t = (1-\theta)\left(p_t - \mathrm{E}_{t-1}[p_t]\right). \tag{*}$$

The aggregate supply curve is thus a price-surprise equation, where prices higher than expected increases output. This is due to the fixed nominal wages, which imply that in such a case the real wage goes down leading to higher output according to (1). It is worth mentioning that $\theta$ plays a role in how strong a price surprise affects output: The higher is $\theta$, the smaller is the output effect. This is because with a higher degree of indexation, a smaller fraction of the real wages will be affected by price changes, and thereby aggregate output will be affected correspondingly less.

(ii) Monetary policy is for simplicity modelled as a matter of setting $p_t$. It is determined by a central bank whose utility function is

$$U = \lambda y_t - \frac{1}{2}\left(p_t - p_{t-1}\right)^2, \qquad \lambda > 0. \tag{3}$$

Derive the utility-maximizing choice of $p_t$ for given price expectations, subject to the aggregate supply schedule derived in (i), and derive the time-consistent, rational-expectations solutions for $y_t$ and $p_t - p_{t-1}$. Discuss.

A Inserting (*) into (3) gives

$$U = \lambda(1-\theta)\left(p_t - \mathrm{E}_{t-1}[p_t]\right) - \frac{1}{2}\left(p_t - p_{t-1}\right)^2,$$

which is maximized w.r.t. $p_t$. The relevant first-order condition is

$$\lambda(1-\theta) - \left(p_t - p_{t-1}\right) = 0,$$

providing immediately the solution for $p_t - p_{t-1}$:

$$p_t - p_{t-1} = \lambda(1-\theta).$$

Taking expectations, one recover

$$\mathrm{E}_{t-1}[p_t] - p_{t-1} = \lambda(1-\theta).$$

Inserting the solutions for $p_t$ and $\mathrm{E}_{t-1}[p_t]$ into (*) immediately gives $y_t = 0$. Hence, in this solution there is an inflation bias $p_t - p_{t-1} = \lambda(1-\theta) > 0$ as the central bank's preferences give it an incentive to push up output. As this is expected by wage setters, wages are set correspondingly high with the end result being no effect on output and too high inflation.

(iii) What is the optimal degree of nominal wage indexation in the economy, i.e., the optimal value of $\theta$, when the central bank's utility function (3) is the welfare measure? Provide an intuitive explanation, and discuss whether the answer would change if the central bank's utility function is quadratic in output and equation (1) is replaced by $y_t = p_t - w_t - \varepsilon_t$, where $\varepsilon_t$ is a supply shock.

A Since the inflation bias is $\lambda(1-\theta)$, and any inflation is costly, it is optimal to have $\theta = 1$. I.e., full wage indexation. The intuition for this result is that with full indexation, higher prices are automatically translated into higher wages, and thus an unchanged real wage. The central bank will thus be unable to affect the real wage and output, and there will for given expectations be *no* benefits of surprise inflation. The central bank will therefore just concentrate on achieving zero inflation. If the central bank has quadratic utility in output, and output is subject to a shock, then a stabilization trade-off will be present in policymaking. In that case, full indexation will be disadvantageous, as it will prevent the central bank from dampening the impact of the shock on output. Some "non-indexation" will be optimal, as this will create some leverage for the central bank to stabilize output. However, the lower is indexation, the higher is the inflation bias, so an optimal degree of indexation will have to weigh the benefits of lower average inflation against the cost of too unstable output.

## QUESTION 3:

**Money-in-the utility function models. Is the timing of utility flows important?**

Consider a model of an economy formulated in discrete time, where representative individuals have utility functions

$$U=\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t},m_{t}\right),\qquad 0<\beta<1, \tag{1}$$

and budget constraints

$$f\left(k_{t-1}\right)+\tau_{t}+\left(1-\delta\right)k_{t-1}+m_{t}=c_{t}+k_{t}+m_{t+1}\left(1+\pi_{t+1}\right), \tag{2}$$

where $c_t$ is consumption, $m_t$ is real money balances at the *beginning* of period $t$, $k_{t-1}$ is physical capital at the end of period $t-1$, $\tau_t$ are monetary transfers by the government, $0<\delta<1$ is capital's rate of depreciation and $\pi_t$ is the inflation rate. The functions $u$ and $f$ are increasing and strictly concave in their arguments. Transfers are financed by money creation only, such that the public budget constraint reads $\tau_t=m_{t+1}\left(1+\pi_{t+1}\right)-m_t$.

(i) Discuss the model. Emphasize the difference with the MIU model from the curriculum where per-period utility arises from *end-of-period* real money.

A The model is a variant of the Money-in-Utility-function models, where a role for money is introduced by having real money providing utility directly (e.g., as a proxy for saved transaction activities in the goods market). In the model of the curriculum, $m_t$ is money held at the end of period $t$. Just as plausible, the current set up assumes that the utility-generating money, $m_t$, is money held in the beginning of period $t$. The budget constraint therefore has on it right-hand side nominal money held in the beginning of period $t+1$ deflated by the period-$t$ price level, leading to the term $M_{t+1}/P_t=m_{t+1}\left(P_{t+1}/P_t\right)=m_{t+1}\left(1+\pi_{t+1}\right)$, where $M_{t+1}$ is nominal money held in the beginning of period $t+1$ and $P_t$ is the price level in period $t$. It is seen from the budget constraint (2) that inflation erodes available resources of the individual—just as in the model of the curriculum (divide (2) through by $\left(1+\pi_{t+1}\right)$).

(ii) Derive the relevant first-order conditions for optimal individual behavior (where transfers are taken as given). For this purpose, use the value function

$$V(k_{t-1}, m_t) = \max \left\{ u(c_t, m_t) + \beta V(k_t, m_{t+1}) \right\},$$

where the maximization is over $c_t$, $m_{t+1}$, $k_t$ subject to (2). [Hint: Simplify the problem by using (2) to substitute out $k_t$ in the value function]

A Using the hint the maximization problem is

$$V(k_{t-1}, m_t) = \max_{c_t, m_{t+1}} \left\{ u(c_t, m_t) + \beta V(k_t, m_{t+1}) \right\},$$

with

$$k_t = f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + m_t - c_t - m_{t+1}(1+\pi_{t+1}).$$

The first-order conditions are then

$$u_c(c_t, m_t) - \beta V_k(k_t, m_{t+1}) = 0, \tag{*}$$

and

$$-(1+\pi_{t+1}) V_k(k_t, m_{t+1}) + V_m(k_t, m_{t+1}) = 0, \tag{**}$$

respectively.

(iii) Interpret the first-order conditions and show that they (along with the expressions for the partial derivatives of the value function derived using the Envelope Theorem) can be combined into the following system:

$$\begin{aligned} u_c(c_t, m_t) &= \beta R_t u_c(c_{t+1}, m_{t+1}), \\ \frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})} &= i_t, \end{aligned}$$

where $R_{t-1} \equiv 1 + f'(k_{t-1}) - \delta$ is the gross real interest rate, and $i_t \equiv R_t(1+\pi_{t+1}) - 1$ is the nominal interest rate.

A (*) shows that consumption in period $t$ is chosen so as to equate the marginal gain of period-$t$ consumption to the discounted utility loss from lesser capital from period $t+1$ and onwards. (**) shows that money left for the next period is chosen so as to balance the marginal losses of less capital with the marginal gain of higher money balances. Using that in optimum

$$V(k_{t-1}, m_t) = u(c_t, m_t) + \beta V(k_t, m_{t+1}),$$

with $c_t$ and $m_{t+1}$ chosen optimally, the Envelope Theorem gives

$$\begin{aligned} V_k(k_{t-1}, m_t) &= \beta V_k(k_t, m_{t+1})(1 + f'(k_{t-1}) - \delta) \\ &= \beta R_{t-1} V_k(k_t, m_{t+1}) \qquad (***) \end{aligned}$$

and

$$V_m(k_{t-1}, m_t) = u_m(c_t, m_t) + \beta V_k(k_t, m_{t+1}). \qquad (****)$$

Forwarding (***) one period and multiplying by $\beta$ gives

$$\beta V_k(k_t, m_{t+1}) = \beta^2 R_t V_k(k_{t+1}, m_{t+2}),$$

which together with (*) give the first of the required expressions:

$$u_c(c_t, m_t) = \beta R_t u_c(c_{t+1}, m_{t+1}).$$

Forwarding (****) one period and dividing through by $V_k(k_t, m_{t+1})$ gives

$$\frac{V_m(k_t, m_{t+1})}{V_k(k_t, m_{t+1})} = \frac{u_m(c_{t+1}, m_{t+1})}{V_k(k_t, m_{t+1})} + \beta \frac{V_k(k_{t+1}, m_{t+2})}{V_k(k_t, m_{t+1})}.$$

Using (*) and $u_c(c_t, m_t) = \beta R_t u_c(c_{t+1}, m_{t+1})$, this reduces to

$$\begin{aligned} 1 + \pi_{t+1} &= \frac{\beta u_m(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} + \frac{1}{R_t}, \\ 1 + \pi_{t+1} &= \frac{\beta u_m(c_{t+1}, m_{t+1})}{\beta R_t u_c(c_{t+1}, m_{t+1})} + \frac{1}{R_t}, \\ (1 + \pi_{t+1}) R_t &= \frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})} + 1, \end{aligned}$$

and finally

$$(1 + \pi_{t+1}) R_t - 1 = \frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})}$$

which by the definition of the nominal interest rate gives the second of the required expressions.

(iv) Is monetary policy steady-state superneutral in the sense that output, $y^{ss} = f(k^{ss})$, is unaffected by inflation? Assess this formally and explain.

A From $u_c(c_t, m_t) = \beta R_t u_c(c_{t+1}, m_{t+1})$ it follows that in steady state,

$$\beta^{-1} = R^{ss} = 1 + f'(k^{ss}) - \delta,$$

which uniquely pins down steady-state capital and output. Monetary policy is thus superneutral, as any changes in money growth, inflation and nominal interest rates does not affect long-run savings incentives (i.e., the long run real interest rate).

(v) Discuss the model's differences and similarities with the MIU model from the curriculum. Emphasize potential differences in the optimal steady-state inflation rate.

A The main difference is that the marginal rate of transformation between money and consumption is $i_t$, and not $I_t \equiv i_t / (1 + i_t)$ as in the case where it is end-of-period money that gives utility. This, however, has no implication for the optimal rate of inflation, as this will be the one that secures a zero nominal interest rate, i.e., implementation of the Friedman rule. This result, and the superneutrality result, are thus unaffected by the modelling of the timing of utility flows of money.